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On the determination of the refractive index of strongly absorbing particles dispersed in a non-absorbing host medium

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Dedicated to Professor Dr Reiner Weichert on the occasion of his 60th birthday

Abstract. This paper is devoted to the development of a theoretical base for the retrieval of optical constants of large strongly absorbing particles dispersed in a non-absorbing host medium. Both single and multiple scattering regimes are studied. Particles can have a spherical shape or be non-spherical, but convex and randomly oriented. Analytical formulae relating Fresnel reflectivities for surfaces of particles to the intensity of the diffused reflected light are proposed. They are based on the single and quasi-single approximations of the radiative transfer theory. The geometrical optics approximation is used to find local optical characteristics (for example the phase function and the extinction coefficient) of turbid media under consideration.

1. Introduction

It is a well known fact that the refractive index of plane surfaces and thin films can be found from measurements of light reflectance [1, 2]. Optical constants of many substances have been obtained in this way [3–5].

The problem is much more complex in the case of disperse media. The Mie theory can be used as a base for the retrieval procedures only in the case of small concentrations of spherical scatterers or for single particles [6]. Things are even more complex for thick disperse media. In this case neither the Fresnel equations nor Mie theory are valid and one should use the radiative transfer equation [7, 8] to relate the diffused intensity of the reflected and transmitted light with optical constants of particles dispersed in a medium under consideration. The particle size distribution, which is not known *a priori*, should be retrieved simultaneously.

The task is simplified if one uses special arrangements for the experiment, which allow the radiative transfer equation to be solved analytically for the problem in question. For example, such a solution exists in case of weakly absorbing optically thick media [7, 8]. The procedure for the retrieval of the refractive index of weakly absorbing particles was proposed in [9]. The special case of densely packed large and small particles in the framework of the six-flux approximation of the radiative transfer theory was studied in [10, 11].

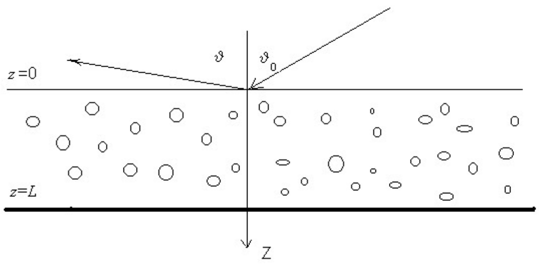

Figure 1. The geometry of the problem.

The task of this paper is to show that there are special spectral regions where familiar techniques based on Fresnel equations [1–5] can be used even in case of particulate media. We shall suppose that the host medium is uniform, isotropic and a non-absorbing one. The idea behind of our approach is simple. It relies on the fact that the Mie theory reduces to Fresnel equations at large values of the parameters $x = 2\pi a/\lambda$ and $c = 8\pi\kappa a/\lambda$, where λ is the wavelength, a is the radius of particles and κ is the imaginary part of the refractive index of scatterers [8, 12]. Thus, our approach can be applied only to large strongly absorbing particles (for example dust aerosols and soils).

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2. Theory

Let us consider a disperse medium illuminated on the top by an infinitely wide beam (see figure 1) with a flux density D_0. The angle of incidence is ϑ_0 and the observation angle is equal to ϑ. We will suppose that the axis OZ of the spherical system of coordinates is directed inwards (see figure 1) and the relative azimuth of the reflected radiation is equal to ϕ. The complex refractive index of particles relative to a host medium is equal to $M = n - \mathrm{i}\kappa$. The refractive index of a host medium and space at $z \leqslant 0$ are equal to n_2 and n_1 respectively. We will suppose that values of n_1, n_2 are real. The surface at $z = L$ (bottom) is totally absorbing.

The intensity I of the reflected diffused light for a plane-parallel layer, presented in figure 1, can be found with the following equation [8, 12]:

$$I = \frac{\hat{p}(\theta) D[1 - \exp(-\alpha\tau_0)]}{4\pi\alpha\mu} \tag{1}$$

where

$$\begin{aligned}
\mu_0 &= \cos(\vartheta_1) \geqslant 0\\
\mu &= \cos(\vartheta_2) \geqslant 0\\
\vartheta_1 &= \arcsin\left(\frac{n_1 \sin(\vartheta_0)}{n_2}\right)\\
\vartheta_2 &= \arcsin\left(\frac{n_1 \sin(\vartheta)}{n_2}\right)\\
\alpha &= \frac{\mu + \mu_0}{\mu\mu_0}\\
D &= (1 - R_{12})(1 - R_{21}) D_0\\
R_{12} &= 0.5(r_1^2 + \rho_1^2)\\
R_{21} &= 0.5(r_2^2 + \rho_2^2)
\end{aligned}$$

and

$$r_1 = \frac{n_1 \cos\vartheta_1 - n_2 \cos\vartheta_0}{n_1 \cos\vartheta_1 + n_2 \cos\vartheta_0} \tag{2}$$

$$\rho_1 = \frac{n_1 \cos\vartheta_0 - n_2 \cos\vartheta_1}{n_1 \cos\vartheta_0 + n_2 \cos\vartheta_1} \tag{3}$$

$$r_2 = \frac{n_2 \cos\vartheta - n_1 \cos\vartheta_2}{n_2 \cos\vartheta + n_1 \cos\vartheta_2} \tag{4}$$

$$\rho_1 = \frac{n_2 \cos\vartheta_2 - n_1 \cos\vartheta}{n_2 \cos\vartheta_2 + n_1 \cos\vartheta}. \tag{5}$$

$\tau_0 = \sigma_{\mathrm{ext}} L$ is the optical thickness of a layer, σ_{ext} is the extinction coefficient, $\hat{p}(\theta) = \omega_0 p(\theta)$, $\omega_0 = \sigma_{\mathrm{sca}}/\sigma_{\mathrm{ext}}$, σ_{sca} is the scattering coefficient, $p(\theta)$ is the phase function and

$$\theta = \arccos\left[-\mu\mu_0 + \sqrt{(1 - \mu^2)(1 - \mu_0^2)} \cos\phi\right]$$

is the scattering angle.

Equation (1) was obtained in the framework of the the single scattering approximation of the radiative transfer theory under the assumption that the optical thickness τ_0 is small [12]. It should be pointed out that only the function $\hat{p}(\theta)$ in equation (1) depends on optical constants of large scatterers. This function can be found from equation (1), measuring values of I and τ:

$$\hat{p}(\theta) = \frac{4\pi\alpha\mu I}{D[1 - \exp(-\alpha\tau_0)]}. \tag{6}$$

Let us show that the phase function is proportional to the Fresnel reflection coefficient for the case under consideration. The phase function $p(\theta)$ for the polydispersions of spherical particles can be found from the following general equation [8]:

$$p(\theta) = \frac{\lambda^2}{\pi} \frac{\int_0^\infty i(a) f(a)\,\mathrm{d}a}{\int_0^\infty C_{\mathrm{sca}} f(a)\,\mathrm{d}a} \tag{7}$$

where $f(a)$ is the particle size distribution (PSD), C_{sca} is the scattering cross section of a single particle and $i(a)$ is the dimensionless Mie intensity, which depends on the refractive index of scatterers, the scattering angle and the size of particles.

The determination of the refractive index from equation (7) is a complex task [6]. It can be simplified in the case of strongly absorbing convex particles with large diffraction $x = \pi d/\lambda$ and absorption $c = 4\kappa x$ parameters. Here κ is the imaginary part of the refractive index, which itself can be small, d is the average size of particles (the diameter in case of spheres) and λ is the wavelength of the incident radiation. The most important feature of such particles at random orientation is that, regardless their specific shape, light scattering patterns of such scatterers are identical to those of spheres [13]. It results from the fact that the normals of any one surface element that assumes a random position are distributed in the same manner as the normals of all surface elements on a sphere [13]. The dimensionless intensities for such scatterers are [13]

$$i = i^r + i^d \tag{8}$$

$$i^r = \frac{x^2}{4} R_F(\theta) \qquad i^d = \frac{x^2 J_1^2(\theta x)}{\theta^2}. \tag{9}$$

Here J_1 is the Bessel function and $R_F(\theta)$ is the Fresnel reflection coefficient

$$\begin{aligned}
R_F &= \frac{|r_p|^2 + |r_s|^2}{2}\\
r_p &= \frac{\cos\psi - M\cos\varphi}{\cos\psi + M\cos\varphi}\\
r_s &= \frac{\cos\varphi - M\cos\psi}{\cos\varphi + M\cos\psi}
\end{aligned} \tag{10}$$

where $\psi = \arcsin(\sin\varphi/M)$ and φ is the angle of incidence of light on the surface of a particle. This angle is related to the scattering angle θ (see figure 2) by the following simple formula [3]:

$$\varphi = \frac{\pi - \theta}{2}. \tag{11}$$

It follows from equations (7)–(9) that

$$p(\theta) = p^d(\theta) + p^r(\theta) \tag{12}$$

where

$$p^d(\theta) = \frac{2\int_0^\infty a^2 J_1^2(ka\theta) f(a)\,\mathrm{d}a}{\omega_0\theta^2 \int_0^\infty a^2 f(a)\,\mathrm{d}a} \tag{13}$$

and

$$p^r(\theta) = \frac{R_F(\theta)}{2\omega_0} \tag{14}$$

where [8] $\omega_0 = 0.5(1+s)$, $s = 0.5\int_0^\pi R_F(\theta)\sin\theta\,\mathrm{d}\theta$ at large values of x, c. To derive equations (13) and (14) we used

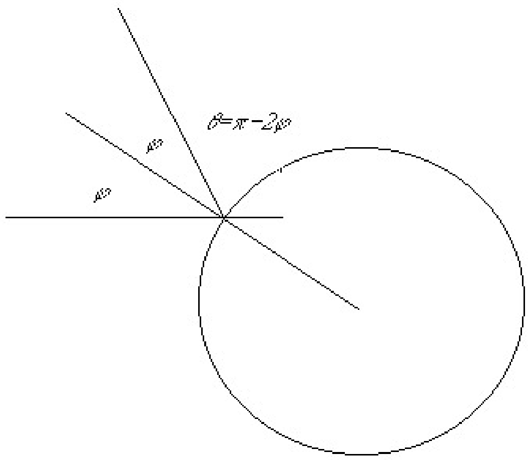

Figure 2. The reflection of light from a particle.

the fact that the extinction cross section of a large particle is equal to $2\pi a^2$ [13]. The integral $p^d(\theta)$ depends on the shape and size of particles and is important only for small scattering angles ($\theta \leqslant \frac{7}{x}$ [13]), which are not considered here. Thus, one obtains, neglecting the diffraction component (see equations (1), (12) and (14))

$$I = \frac{R_F(\theta) D[1 - \exp(-\alpha\tau_0)]}{8\pi\alpha\mu}. \tag{15}$$

To study the applicability of the phase function (14) to the real-world particles we plotted the phase functions of dust grains obtained with the use of equation (14) and the Mie theory in figure 3. Calculations were performed at $\lambda = 0.55$ μm. The refractive index of dust aerosols was assumed to be $1.53 - 0.008$i at this wavelength. The particle size distribution $f(a)$ in the Mie calculations was

$$f(a) = Ca^6 \exp(-9a/a_{\mathrm{ef}}) \tag{16}$$

where a_{ef} is the effective radius and C is the normalization constant ($\int_0^\infty f(a)\,\mathrm{d}a = 1$). The effective radius of particles

$$a_{\mathrm{ef}} = \frac{\int_0^\infty a^3 f(a)\,\mathrm{d}a}{\int_0^\infty a^2 f(a)\,\mathrm{d}a} \tag{17}$$

was equal to 5, 15 and 30 μm.

The interesting feature of figure 3 is that even comparatively small dust particles with $a_{\mathrm{ef}} = 5$ μm can be considered as strongly absorbing (in the sense that simple equation (14) can be applied) in the broad region of angles in the backward hemisphere ($\theta = 100$–150 degrees). Thus, these angles are favourable ones from the point of view of the inverse problem solution. The important conclusion is that one does not need to have exact information about either the shape of particles or about their size in this particular case.

From equation (15) it follows

$$R_F(\theta) = \frac{8\pi\alpha\mu I}{D[1 - \exp(-\alpha\tau_0)]}. \tag{18}$$

Equation (18) reduces the complex problem of the determination of the refractive index of scatterers to the standard problem of the reflectometry of uniform surfaces. The value of the Fresnel reflection coefficient for the surface of a particle can be obtained from measurements of values of I and τ_0.

Equation (15) can be generalized for the case of incident polarized radiation:

$$\boldsymbol{I} = A\hat{P}(\theta)\boldsymbol{I}_0 \tag{19}$$

where, neglecting for the simplicity differences $n_1 - n_2$, one obtains:

$$A = \frac{D_0[1 - \exp(-\alpha\tau_0)]}{8\pi\alpha\mu}. \tag{20}$$

Here $\boldsymbol{I}$ is the Stokes vector of the reflected light, $\boldsymbol{I}_0$ is the Stokes vector of the incident light, normalized to the value of the density of the incident flux, and $\hat{P}(\theta) = \hat{L}(\pi - j_2)\hat{R}_F(\theta)\hat{L}(-j_1)$, where matrices $\hat{R}_F(\theta)$, $\hat{L}(j)$, are presented in appendix A.

3. Account for multiple light scattering

The same approach can be applied in the case of multiple scattering of photons in disperse media (any values of τ_0). To this end one should make the following transfer in the equations of the previous section (see appendix B):

$$\omega_0 \rightarrow \frac{\omega_0}{1 - \omega_0 F} \qquad \tau_0 \rightarrow \tau_0(1 - \omega_0 F) \tag{21}$$

where $F = 1 - v\int_{\pi/2}^{\pi} R_F(\theta)\sin\theta\,\mathrm{d}\theta$ and $v = 1/2(1+s)$. The result for the diffused intensity I (see equation (15)) is

$$I = \frac{R_F(\theta)D}{8\pi\alpha\mu(1 - \omega_0 F)}\{1 - \exp[-\alpha\tau_0(1 - \omega_0 F)]\}. \tag{22}$$

This approximation is valid at any τ_0. It is called the quasi-single scattering approximation of the radiative transfer theory [7, 8].

It is easier to deal with optically thick media ($\tau_0 \rightarrow \infty$) while carrying out experiments. The intensity of light reflected from such media is larger (see equation (22)) and one does not need to have information on the value of τ_0 in this case to find optical constants. Thus, the main equation for the inverse problem solution in the multiple scattering regime is (see equation (22) at $\tau_0 \rightarrow \infty$)

$$R(\mu_0, \mu, \phi) = \frac{\gamma R_F(\theta)}{4(\mu + \mu_0)} \tag{23}$$

where

$$\theta = \arccos\left[-\mu\mu_0 + \sqrt{(1-\mu^2)(1-\mu_0^2)}\cos(\phi)\right]$$

$$\gamma = \frac{1}{2 - (1+s)F}$$

$$R(\mu, \mu_0, \phi) = \frac{\pi I}{\mu_0 D}$$

is the reflection function of a layer. The comparisons of equation (23) and the solution of the radiative transfer equation (see appendix B) at the normal illumination of a layer are presented in figure 4. One can see that the error of equation (23) is small (0.5–2% at $\vartheta \leqslant 75°$). Calculations

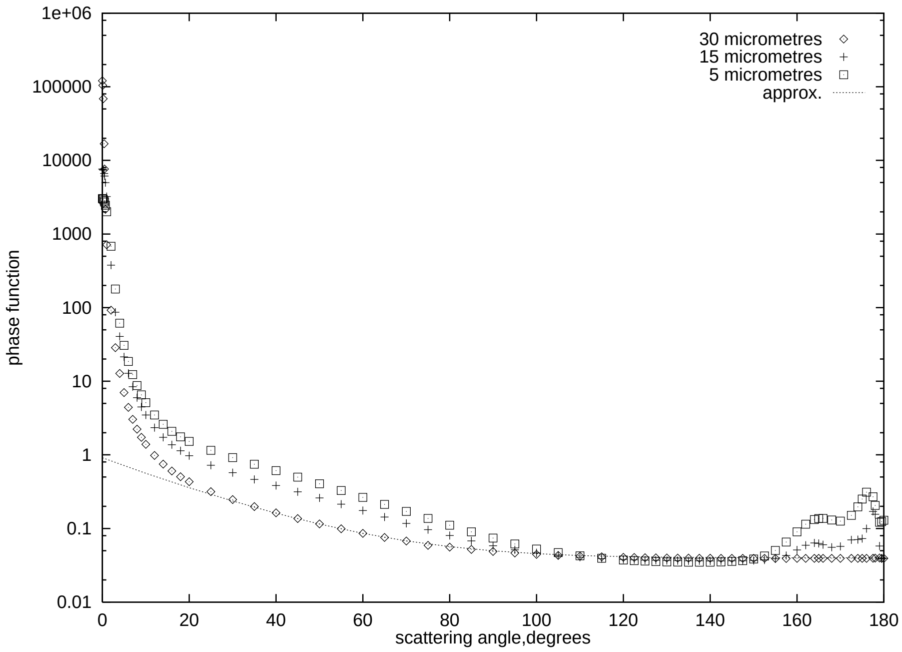

Figure 3. Phase functions of dust particles with different values of the effective radius equal to 5, 15 and 30 μm, calculated with the Mie theory and approximate equation (14) at $\lambda = 0.55$ μm, $n = 1.53$, $\kappa = 0.008$ for the gamma particle size distribution (16).

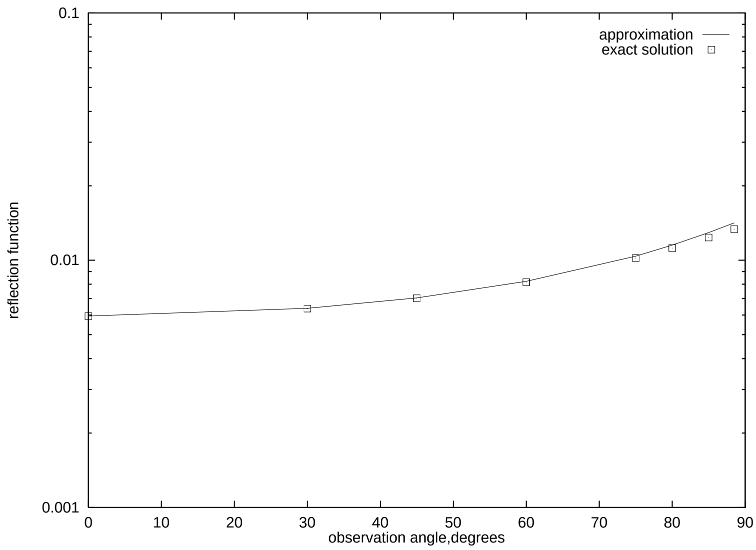

Figure 4. Reflection function of a plane-parallel layer with the optical thickness equal to 10 at $n = 1.53$, $\kappa = 0.008$, $a_{\mathrm{ef}} = 30$ μm for the gamma particle size distribution (16), calculated with the exact radiative transfer code and equation (23) at the normal illumination.

were performed at $a_{\mathrm{ef}} = 30$ μm, $\tau_0 = 10$. However, the accuracy of equation (23) does not change considerably in the range of $\tau_0 \geqslant 5$.

Thus the principal conclusion which can be derived from our calculations is that there is a possibility of the retrieval of the Fresnel reflectivities $R_F(\theta)$ (and, therefore, the refractive index of large strongly absorbing particles) from measurements of the reflection function (see equation (23)) of a turbid layer. Precise information about the size or shape of grains and the optical thickness of a layer is not required. From these measurements one can easily find the parameter $y = \gamma R_F(\theta)$, namely (see equation (23))

$$y = 4(\mu + \mu_0)R(\mu, \mu_0, \phi) \tag{24}$$

where $R(\mu, \mu_0, \phi)$ is the measured reflection function. Note that the value of s is rather small and $F \approx 1$, $\gamma \approx 1$ (see table 1). Thus, the parameter γ does not influence very much on the value of $y \approx R_F(\theta)$.

4. Conclusion

We have managed to reduce the complex problem of the reflectometry of disperse media to the same problem for a uniform surface, which is characterized by the Fresnel coefficient $R_F(\theta)$. This surface is a surface of a particle. The value of $R_F(\theta)$ can be obtained from measurements of the intensity of light, reflected from a turbid layer. Equations

Table 1. Dependence of s, F and γ on n.

n	s	F	γ
1.1	0.025	0.999	1.025
1.2	0.044	0.995	1.040
1.333	0.066	0.989	1.057
1.4	0.077	0.985	1.065
1.5	0.092	0.979	1.074
2.0	0.0161	0.943	1.105

(15) and (22), which relate intensity of the reflected light from a layer with the Fresnel reflection coefficient for the surface of a particle, can be used only in the case of large absorption of light by particles and their large size in comparison with the wavelength. This is the main shortcoming of the method proposed. The attractive point of the method is the simplicity of the main equations (15) and (22).

The methods for the retrieval of the refractive index M from the value of $R_F(\theta)$ will not be discussed here, because this is a well studied subject with a long history [1].

It should be pointed out that spectral measurements of the reflection function can be used to get information on the spectral dependences of the refractive indices of scatterers [14, 15].

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Appendix A.

We present here formulae for the Fresnel and rotation matrices, which appeared in equation (19) [8, 12]:

$$\hat{R}_F = \frac{1}{2}\begin{pmatrix} r_p r_p^* + r_s r_s^* & r_p r_p^* - r_s r_s^* & 0 & 0 \\ r_p r_p^* - r_s r_s^* & r_p r_p^* + r_s r_s^* & 0 & 0 \\ 0 & 0 & 2\,\mathrm{Re}(r_p r_s^*) & 2\,\mathrm{Im}(r_p r_s^*) \\ 0 & 0 & -2\,\mathrm{Im}(r_p r_s^*) & 2\,\mathrm{Re}(r_p r_s^*) \end{pmatrix}$$

$$\hat{L}(\chi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\chi & \sin 2\chi & 0 \\ 0 & -\sin 2\chi & \cos 2\chi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where the reflectances r_p, r_s are defined in equation (10) and

$$\cos j_1 = \frac{-\mu_0 + \mu\cos\theta}{(-1)^l\sqrt{(1-\cos^2\theta)(1-\mu^2)}}$$

$$\cos j_2 = \frac{-\mu + \mu_0\cos\theta}{(-1)^l\sqrt{(1-\cos^2\theta)(1-\mu_0^2)}}$$

where $l = -1$ at $\phi \geqslant \pi$ and $l = 1$ otherwise.

Appendix B. Quasi-single scattering approximation

Let us consider now the derivation of the quasi-single scattering approximation (see equation (22)) from the radiative transfer equation (RTE) for the diffused intensity $I(\mu_0, \mu, \phi)$ of light reflected from a multiple light scattering plane-parallel layer [7, 8]:

$$\mu\frac{\mathrm{d}I(\mu_0,\mu,\phi)}{\mathrm{d}z} = -\sigma_{\mathrm{ext}}I(\mu_0,\mu,\phi) + \frac{\sigma_{\mathrm{sca}}}{4\pi}\int_0^1 \mathrm{d}\mu^*\int_0^{2\pi}\mathrm{d}\phi^*\, p(\theta^*)I(\mu_0,\mu^*,\phi^*) + B \tag{B.1}$$

where

$$B = \frac{\omega_0 D_0 p(\theta)\exp(-\tau_0/\mu_0)}{4\pi}$$

is the source function, τ is the optical depth and σ_{ext} and σ_{sca} are extinction and scattering coefficients

$$\theta^* = \arccos\Big[\mu\mu^* + \sqrt{(1-\mu^2)(1-\mu^{*2})}\cos(\phi-\phi^*)\Big]$$

$$\theta = \arccos\Big[-\mu\mu_0 + \sqrt{(1-\mu^2)(1-\mu_0^2)}\cos(\phi)\Big]$$

μ_0 and μ are the cosines of the incident ϑ_0 and observation ϑ angles, ϕ is the relative azimuth of the reflected radiation and z is the geometrical depth, D_0 is the net flux of the incident light per unit area, oriented normally to the parallel light beam. We assume that refractive indices of a host medium and space around a layer coincide. The boundary conditions for this equation are [8]

$$I(\mu_0,\mu,\phi) = 0 \qquad \vartheta_0 \leqslant \pi/2 \qquad \tau = 0 \tag{B.2}$$

$$I(\mu_0,\mu,\phi) = 0 \qquad \vartheta_0 \geqslant \pi/2 \qquad \tau = \tau_0 \tag{B.3}$$

where τ_0 is the optical thickness of a layer. They state that there is no diffuse light which comes to the boundaries of a layer from outside.

For media with large strongly absorbing particles, which are the only two considered here, the RTE can be approximately solved, assuming that [7]

$$p(\theta) = F\delta(\theta) + (1-F)p^*(\theta) \tag{B.4}$$

where $\delta(\theta)$ is the delta function, $p^*(\theta) = p^r(\theta)/(1-F)$ and the value of F represents the flux scattered in the forward hemisphere

$$F = 1 - \frac{1}{2(1+s)}\int_{\pi/2}^{\pi} R_F(\theta)\,\sin\theta\,\mathrm{d}\theta. \tag{B.5}$$

The main support for this approximation comes from the fact that photons scattered in the forward hemisphere have almost no chance of reaching a receiver situated above the scattering layer (for strongly absorbing media with strongly forward-peaked phase functions; see figure 3). After substitution of equation (B.4) into equation (1) one obtains

$$\mu\frac{\mathrm{d}I(\mu_0,\mu,\phi)}{\mathrm{d}z} = -(\sigma_{\mathrm{ext}} - \sigma_{\mathrm{sca}}F)I(\mu_0,\mu,\phi) + \frac{(1-F)\sigma_{\mathrm{sca}}}{4\pi}\int_0^1 \mathrm{d}\mu^*\int_0^{2\pi}\mathrm{d}\phi^*\, p(\theta^*)I(\mu_0,\mu^*,\phi^*) + B \tag{B.6}$$

or

$$\mu \frac{\mathrm{d}I(\mu_0, \mu, \phi)}{\mathrm{d}\tau^*} = -I(\mu_0, \mu, \phi) + \frac{\omega_0^*}{4\pi} \int_0^1 \mathrm{d}\mu^* \int_0^{2\pi} \mathrm{d}\phi^* \, p^*(\theta^*) I(\mu_0, \mu^*, \phi^*) + B^* \tag{B.7}$$

where

$$\tau^* = (1 - \omega_0 F)\tau \tag{B.8}$$

$$\omega_0^* = \frac{\omega_0(1-F)}{1-\omega_0 F} \qquad B^* = \frac{\omega_0 D_0 p^r(\theta) \exp(-\tau^*/\mu_0)}{4(1-\omega_0 F)} \tag{B.9}$$

and $\tau = \sigma_{\text{ext}} z$ is the optical depth. The integral term in equation (B.7) can be neglected because the value of $\omega_0^* p^*(\theta^*) \ll 1$ at most scattering angles in the backward hemisphere (see figure 3). Thus, it follows from equations (B.7) and (B.9) that

$$\mu \frac{\mathrm{d}I(\mu_0, \mu, \phi)}{\mathrm{d}\tau^*} = -I(\mu_0, \mu, \phi) + \frac{\omega_0 D_0 p^r(\theta) \exp(-\tau^*/\mu_0)}{4(1-\omega_0 F)}. \tag{B.10}$$

The formal solution of this differential equation can be written in the following form

$$I(\mu_0, \mu, \phi) = \frac{1}{\mu} \int_0^{\tau_0^*} \mathrm{d}\tau' \, B(\tau') \exp(-\tau'/\mu) \tag{B.11}$$

where $\tau_0^* = (1 - \omega_0 F)\tau_0$ and

$$B(\tau') = \frac{\omega_0 D_0 p^r(\theta) \exp(-\tau'/\mu_0)}{4(1-\omega_0 F)}. \tag{B.12}$$

After the substitution of equation (12) into equation (11) one obtains the following approximate formula for the reflection function $R(\mu_0, \mu, \phi) = \pi I(\mu_0, \mu, \phi)/\mu_0 D_0$

$$R(\mu_0, \mu, \phi) = \frac{\omega_0 p^r(\theta)\{1 - \exp[-\tau_0^*(1/\mu_0 + 1/\mu)]\}}{4(1-\omega_0 F)(\mu_0 + \mu)} \tag{B.13}$$

or

$$R(\mu_0, \mu, \phi) = \frac{\gamma R_F(\theta)\{1 - \exp[-\tau_0^*(1/\mu_0 + 1/\mu)]\}}{4(\mu + \mu_0)} \tag{B.14}$$

where

$$\gamma = \frac{1}{2 - (1+s)F}.$$

This is the well-known solution of the RTE in the quasi-single scattering approximation.

Thus, the substitution of the diffraction term in the RTE by the delta function allows us to obtain the approximate analytical solution of the radiative transfer equation. This simple equation can be used to avoid complex calculations with equation (1) and to simplify the inverse problem solution for multiple light scattering media with large strongly absorbing particles.

Appendix C. Symbols and abbreviations

a	radius of particles
a_{ef}	effective radius
c	absorption parameter
$f(a)$	particle size distribution
i	dimensionless light intensity
C_{sca}	scattering cross section
D_0	density of the incident flux
F	light flux in a forward hemisphere
I	intensity of diffused light
L	rotation matrix
$M = n - \mathrm{i}\kappa$	relative refractive index of particles
R_F	Fresnel matrix
n_1	refractive index of space outside a disperse layer
n_2	refractive index of a host medium
$p(\theta)$	phase function
$R(\mu, \mu_0, \phi)$	reflection function
R_F	Fresnel reflection coefficient
θ	scattering angle
ϑ	observation angle
ϑ_0	incident angle
λ	wavelength
μ	cosine of the observation angle
μ_0	cosine of the incident angle
x	size parameter
σ_{sca}	scattering coefficient
σ_{ext}	extinction coefficient
τ	optical depth
τ_0	optical thickness
ϕ	azimuth
ω_0	single scattering albedo

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